A Unified Approach to the Design, Measurement, and Tuning of Coupled-Resonator Filters

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***Abstract*— The concept of coupling coefficients has been a very useful one in the design of small-to-moderate bandwidth microwave filters. It is shown in this paper that the group delay of the input reflection coefficients of sequentially tuned resonators contains all the information necessary to design and tune filters, and that the group-delay value at the center frequency of the filter can be written quite simply in terms of the low-pass prototype values, the *LC* elements of a bandpass structure, and the coupling coefficients of the inverter coupled filter. This provides an easy method to measure the key elements of a filter, which is confirmed by results presented in this paper. It is also suggested that since the group delay of the reflection coefficient (i.e., the time taken for energy to get in and out of the coupled resonators) is easily measured, it is a useful conceptual alternative to coupling concepts.**

***Index Terms*—Coupling, filters, group delay, resonators.**

I. Introduction

THE DESIGN of microwave filters is based on well-established techniques with perhaps the lowpass–bandpass–inverter coupled-resonator process being the most common design method [1]. For small-to-moderate bandwidth filters, the concept of coupled resonators to realize a particular response has a sound mathematical and practical basis. Once the resonant cavities have been selected, only the coupling values need to be set to generate the filter response. By employing cross couplings between nonadjacent resonators, linear phase and generalized Chebyshev responses can be obtained. Thus, synchronously tuned resonator cavities which are coupled appropriately can realize most of the filter characteristics required even for very demanding specifications.

The calculation of coupling values can be obtained from the literature for a considerable range of microwave resonators, and numerical techniques using commercial software are now available which can be applied to three-dimensional structures. Even so, due to the large variety of resonators and coupling configurations, as well as the approximations often applied to simplify analysis, it is still often necessary to empirically determine coupling values. There are several methods for determining coupling values, but the reflection technique is particularly useful for *in situ* filter measurements. Furthermore, with the measuring capability of vector network analyzers, filter tuning can now be done in a precise convergent way. The alternating short-circuit technique of Dishal [2] is modified here to include directly setting the coupling values as well as the resonant frequency and "real time" adjustment for the effects of frequency pulling of the resonant frequency. Setting the coupling values arises from the simple relationship between the coupling and group delay of the reflected signal. The relevant equations are derived in this paper and confirmed by measured results on filters and by comparison with other techniques.

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II. Theory

The standard approach to filter design using low-pass $\rightarrow$ bandpass $\rightarrow$ inverter coupled resonators is shown in Fig. 1. The nomenclature used follows convention. The coupling between the connecting lines and the input and output resonators, referred to as the external Q or Q_E, and the coupling between resonators k_{ij} are readily specified in terms of the prototype g values and the relative bandwidth. Once these coupling values are set and the resonators all tuned to the center frequency, the inverter coupled filter will have the frequency response predicted by the transformed low-pass prototype within the accuracy limits of the frequency transformation and the representation of the physical circuit by lumped elements.

Q_E and k_{ij} can be determined from the reflected signal S_{11} as successive resonators are tuned to resonance. These parameters are related by quite simple equations to the phase- and group-delay response of S_{11}. The group delay of S_{11} is defined as

$$\Gamma_d(\omega) = -\frac{\delta\phi}{\delta\omega} \tag{1}$$

where $\phi =$ phase of S_{11} (rad) and $\omega =$ angular frequency

For the bandpass circuit of Fig. 1, the group delay of S_{11} can be calculated directly from the equivalent circuit or from the low-pass prototype. The calculation from the low-pass prototype enables the group delay to be expressed directly in terms of the normalized g values and the bandwidth of the bandpass filter. In this case,

$$\Gamma_d(\omega) = -\frac{\partial\phi}{\partial\omega^1}\,\frac{\partial\omega^1}{\partial\omega} \tag{2}$$

where $\phi =$ phase of S_{11} (rad) for the low-pass prototype and $\omega^1 =$ angular frequency of low-pass prototype.

For the standard low-pass to bandpass transformation

$$\omega^1 \rightarrow \frac{\omega_0}{(\omega_2 - \omega_1)}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \tag{3}$$

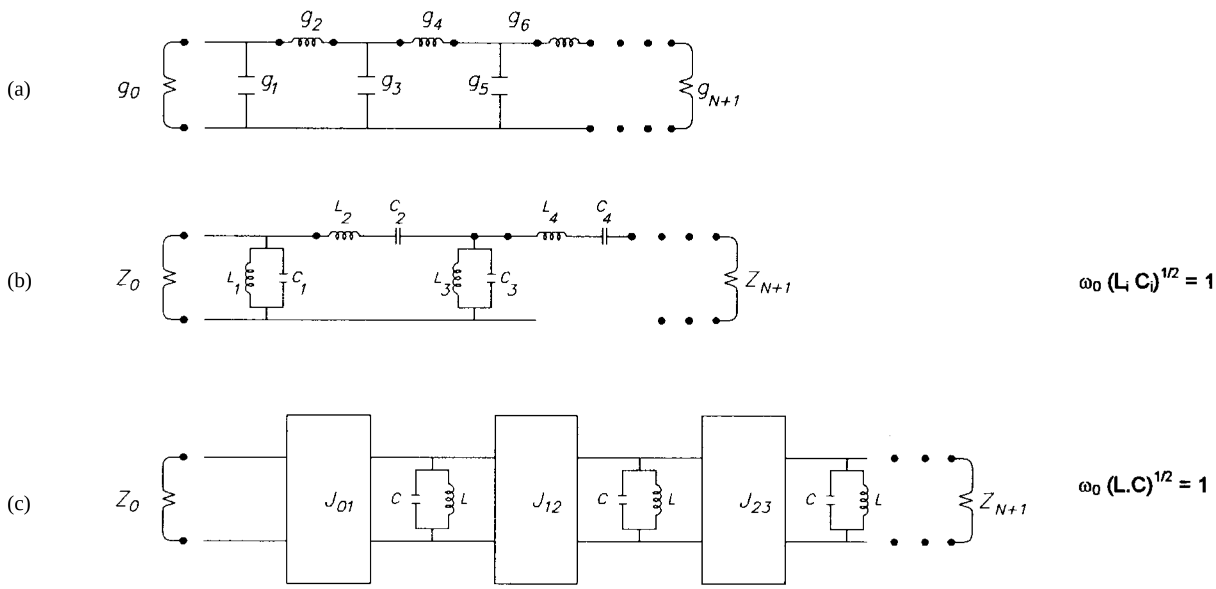

Fig. 1. Circuit elements for (a) low-pass, (b) bandpass, and (c) inverter coupled-filter structures.

where $\omega_0 =$ center frequency of bandpass filter, $\omega_1 =$ lower frequency edge of bandpass filter, and $\omega_2 =$ upper frequency edge of bandpass filter. $\omega_0, \omega_1, \omega_2$ can be defined somewhat differently depending on the type of filter response. For most microwave filters, the Chebyshev response definitions are generally applicable. In this case, $\omega_0 = (\omega_1\ \omega_2)^{1/2}$ and $\omega_c = 1$ (the low-pass cutoff frequency). ω_1 and ω_2 are defined as the lower and upper extremities of the in-band equiripple response.

The group delay of S_{11} for the bandpass circuit and for the inverter coupled circuit is then given by

$$\begin{aligned}\Gamma_d(\omega) &= -\frac{\omega_0}{(\omega_2-\omega_1)}\left(\frac{1}{\omega_0}+\frac{\omega_0}{\omega^2}\right)\frac{\delta\phi}{\delta\omega^1}\\ &= -\frac{\omega^2+\omega_0^2}{\omega^2(\omega_2-\omega_1)}\,\frac{\delta\phi}{\delta\omega^1}.\end{aligned} \tag{4}$$

Now

$$S_{11} = \frac{Z_{\rm IN}-Z_0}{Z_{\rm IN}+Z_0}$$

and, for the lossless case where $Z_{\rm IN}$ is purely imaginary and Z_0 is real, then

$$S_{11} = \frac{jX_{\rm IN}-Z_0}{jX_{\rm IN}+Z_0}.$$

Therefore,

$$\begin{aligned}\phi &= -\tan^{-1}\frac{X_{\rm IN}(\omega)}{Z_0} - \tan^{-1}\frac{X_{\rm IN}(\omega)}{Z_0}\\ &= -2\tan^{-1}\frac{X_{\rm IN}(\omega)}{Z_0}\end{aligned} \tag{5}$$

$$\therefore\ \Gamma_d(\omega) = \frac{2(\omega^2+\omega_0^2)}{\omega^2(\omega_2-\omega_1)}\,\frac{\delta}{\delta\omega^1}\,\tan^{-1}\frac{X_{\rm IN}(\omega^1)}{Z_0}. \tag{6}$$

Consider the first single element of Fig. 1, which in this case is the shunt capacitor g_1 of the normalized low-pass prototype, shown in (7), at the bottom of the page. (The remaining elements are disconnected from g_1.) At the center frequency, $\omega = \omega_0$

$$\therefore \Gamma_{d_1}(\omega_0) = \frac{4g_0g_1}{(\omega_2-\omega_1)}. \tag{8}$$

$$\begin{aligned}X_{\rm IN} &= -\frac{1}{\omega^1 g_1}\\ Z_0 &= g_0\\ \tan^{-1}\frac{X_{\rm IN}(\omega^1)}{Z_0} &= \tan^{-1} -\frac{1}{\omega^1 g_1 g_0}\\ \frac{\delta}{\delta\omega^1}\tan^{-1}\left(-\frac{1}{\omega^1 g_1 g_0}\right) &= \frac{g_0g_1}{1+(g_0g_1\omega^1)^2}\\ \therefore\ \Gamma_d(\omega) &= \frac{2(\omega^2+\omega_0^2)}{\omega^2(\omega_2-\omega_1)}\,\frac{g_0g_1}{1+(g_0g_1\omega^1)^2}\\ &= \frac{2(\omega^2+\omega_0^2)g_0g_1}{\omega^2(\omega_2-\omega_1)\left(1+(g_0g_1)^2\left(\frac{\omega_0}{\omega_2-\omega_1}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\right)^2\right)}\end{aligned} \tag{7}$$

	Low Pass Prototype	**Band Pass Filter**	**Inverter Coupled Filter**
n = 1	$\Gamma_{d1} = \dfrac{4\,g_0\,g_1}{\Delta\omega}$	$\Gamma_{d1} = 4C_1\,Z_0$	$\Gamma_{d1} = \dfrac{4Q_E}{\omega_0}\,;\,Q_E = \dfrac{g_0\,g_1\,\omega_0}{\Delta\omega}$
n = 2	$\Gamma_{d2} = \dfrac{4\,g_2}{g_0\,\Delta\omega}$	$\Gamma_{d2} = \dfrac{4\,L_2}{Z_0}$	$\Gamma_{d2} = \dfrac{4}{\omega_0\,Q_E\,k_{12}^2}\,;\,k_{12} = \dfrac{\Delta\omega}{\omega_0\,(g_1\,g_2)^{1/2}}$
n = 3	$\Gamma_{d3} = \dfrac{4\,g_0\,(g_1+g_3)}{\Delta\omega}$	$\Gamma_{d3} = 4(C_1+C_3)\,Z_0$	$\Gamma_{d3} = \Gamma_{d1} + \dfrac{4\,Q_E\,k_{12}^2}{\omega_0\,k_{23}^2}\,;\,k_{23} = \dfrac{\Delta\omega}{\omega_0\,(g_2\,g_3)^{1/2}}$
n = 4	$\Gamma_{d4} = \dfrac{4\,(g_2+g_4)}{g_0\,\Delta\omega}$	$\Gamma_{d4} = \dfrac{4\,(L_2+L_4)}{Z_0}$	$\Gamma_{d4} = \Gamma_{d2} + \dfrac{4\,k_{23}^2}{\omega_0\,Q_E\,k_{12}^2\,k_{34}^2}\,;\,k_{34} = \dfrac{\Delta\omega}{\omega_0\,(g_3\,g_4)^{1/2}}$
n = 5	$\Gamma_{d5} = \dfrac{4\,g_0\,(g_1+g_3+g_5)}{\Delta\omega}$	$\Gamma_{d5} = 4\,(C_1+C_3+C_5)\,Z_0$	$\Gamma_{d5} = \Gamma_{d3} + \dfrac{4\,Q_E\,k_{12}^2\,k_{34}^2}{\omega_0\,k_{23}^2\,k_{45}^2}\,;\,k_{45} = \dfrac{\Delta\omega}{\omega_0\,(g_4\,g_5)^{1/2}}$
n = 6	$\Gamma_{d6} = \dfrac{4\,(g_2+g_4+g_6)}{g_0\,\Delta\omega}$	$\Gamma_{d6} = \dfrac{4\,(L_2+L_4+L_6)}{Z_0}$ $\Delta\omega = \omega_2 - \omega_1$	$\Gamma_{d6} = \Gamma_{d4} + \dfrac{4\,k_{45}^2\,k_{23}^2}{\omega_0\,Q_E\,k_{12}^2\,k_{34}^2\,k_{56}^2}\,;\,k_{56} = \dfrac{\Delta\omega}{\omega_0\,(g_5\,g_6)^{1/2}}$
	(a)	(b)	(c)

Fig. 2. Group-delay values at ω_0 in terms of (a) low-pass prototype, (b) bandpass, and (c) inverter coupled-filter elements.

Exactly the same result can be obtained by considering the input impedance of the single bandpass resonator (with all other resonators disconnected). Alternatively, the impedance and frequency scaled values for the input resonator can be inserted in (7) to give

$$\Gamma_{d1}(\omega_0) = 4C_1 Z_0 \tag{9}$$

and, in terms of the inverter coupled circuit,

$$\Gamma_{d_1}(\omega_0) = \frac{4Q_E}{\omega_0}. \tag{10}$$

Now, consider two elements with the second element g_2 shorted to ground. In terms of the low-pass, bandpass, and inverter coupled circuit, the group delay of S_{11} can be calculated at the center frequency ω_0 as

$$\Gamma_{d_2}(\omega_0) = \frac{4g_2}{g_0(\omega_2 - \omega_1)} \tag{11}$$

$$\Gamma_{d_2}(\omega_0) = \frac{4L_2}{Z_0} \tag{12}$$

$$\Gamma_{d_2}(\omega_0) = \frac{16}{\omega_0^2 k_{12}^2 \Gamma_{d_1}(\omega_0)} \tag{13}$$

where k_{12} is the coupling coefficient between resonators 1 and 2.

This process can be repeated as each element or resonant circuit is added into the network. Note that if the dual circuit is used (i.e., g_1 is a series inductor), the same equations apply except that CZ_0 is replaced by L/Z_0 and vice versa in the bandpass structure. Note that for the bandpass filter, the group delay determines the value of the odd-numbered capacitors and the even-numbered inductors. The other values are determined directly from the resonance condition $\omega_0^2 L_i C_i = 1$.

A summary of the relevant equations is given in Fig. 2 for up to six resonators. The extension to higher numbers of resonators is obvious.

Fig. 3 shows the frequency response of Γ_d around the center frequency ω_0 and also illustrates the frequency values for the 0° and 180° phase crossings as each successive resonator is tuned to resonance. The coupling values between resonators can be calculated from the frequency values at the 0° and 180° crossings of the phase of S_{11}, as shown by MacDonald [3] and Atia and Williams [4]. A reflection-phase tuning method based on Dishal's alternating short approach has been reported in the literature [5], but this is rather slow and tedious to apply. A computer-controlled measurement of coupling parameters using the frequency-crossing method which allows interactive measurement and tuning has also been developed [6].

It is interesting to note that at ω_0 the group delay of S_{11} is determined only by the shunt elements for an odd number of resonators and by the series elements for an even number of resonators. The reverse would apply if the dual circuit had been used. Making a filter to generate a particular response is, in principle, simply done by setting the group delay at ω_0 to the values determined from the low-pass prototype equation of Fig. 2 and maintaining a symmetrical response as each resonator is successively coupled into the circuit.

The equations given in Fig. 3 can also be used to measure the coupling values in the filter structure. In general, the zero-crossing technique should give the more accurate measurement since it is the frequency only being measured. The capability of vector analyzers to measure the group delay of S_{11} and the precision of the measurement enable filters to be measured and tuned precisely and noniteratively to realize any filter functions which can be derived from the low-pass prototypes shown.

The alternating short tuning technique of Dishal has been a standard tuning technique, but it provides no information

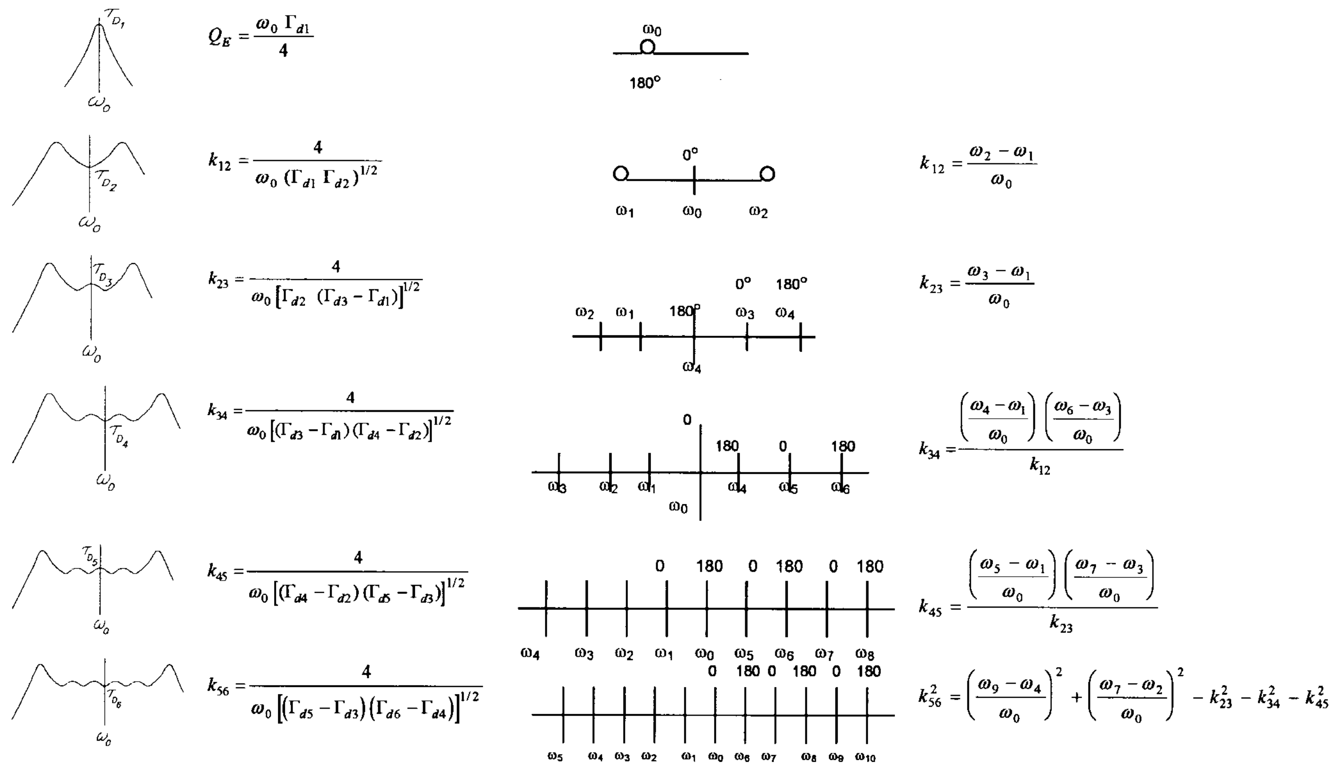

Fig. 3. Coupling values in terms of group-delay and frequency-crossing parameters.

TABLE I
NORMALIZED GROUP-DELAY VALUES FOR 0.01-dB RIPPLE CHEBYCHEV FILTERS

(ns x MHz)	n 3	4	5	6	7	8
$\overline{\Gamma}_{d1}$	400.5	453.8	481.5	497.4	507.3	513.9
$\overline{\Gamma}_{d2}$	617.5	764.1	830.7	865.8	886.4	899.5
$\overline{\Gamma}_{d3}$		1294.9	1485.6	1573	1620.2	1648.6
$\overline{\Gamma}_{d4}$			1661.5	1843	1926.1	1971.2
$\overline{\Gamma}_{d5}$				2526	2773.1	2828.2
$\overline{\Gamma}_{d6}$					2812.5	3002
$\overline{\Gamma}_{d7}$						3818.4

about the coupling values and does not allow for the detuning of the previous resonator as the next resonator is tuned to resonance. Consequently, iterative tuning is typically required, and where the inter-resonator couplings have to be adjusted to realize a very precise response or compensate for design or manufacturing tolerances, the number of iterations can be quite high. By setting the group-delay values at ω_0 and keeping the group-delay response symmetrical about ω_0 as each resonator is successively tuned, both of these drawbacks are removed.

Table I tabulates normalized group-delay values for a 0.01-dB Chebyshev filter from three to eight sections. To realize a 0.01-dB ripple Chebyshev filter of bandwidth Δf (in megahertz), the normalized group-delay values are divided by Δf (in megahertz). As each resonator is tuned to the center frequency and the group delay of S_{11} successively set to the calculated values, the filter will then generate the required Chebyshev response. Note that the group-delay values are independent of the center frequency.

III. MEASUREMENT

To illustrate the application of the above equations, a typical filter design will be considered. The procedure is applicable to any general filter structure provided that the actual resonators and coupling networks are accurately modeled by *LC* networks over the frequency range of interest.

A conventional Chebyshev filter was required with the following characteristics:

center frequency	2.3 GHz;
ripple	0.01 dB;
equiripple bandwidth	26.9 MHz;
number of resonators	6.

TABLE II
DESIGN VALUES FOR STANDARD CHEBYSHEV FILTER

Resonator	Q_E or k_{ij}	Γ_d (ns)	Measured Coupling (Frequency Crossings)
1	66.8	18.5	
1, 2	.01135	32.2	.00112
1, 2, 3	.00771	58.5	.00772
1, 2, 3, 4	.00726	68.5	.00724
1, 2, 3, 4, 5	.00771	93.9	.00775
1, 2, 3, 4, 5, 6	.01135		
6	66.8		

TABLE III
PREDICTED AND MEASURED RESULTS FOR FILTER

	Predicted	Measured
Minimum Return Loss	26.3 dB	26 dB
Equi Ripple Bandwidth	26.9	26.9* MHz
Transmission Group Delay at 2.3 GHz	44.6	44.7 ns
Insertion Loss at 2.3 GHz	2 dB	1.9 dB

*In the presence of loss, the equiripple bandwidth is not clearly defined. The bandwidth was determined from the group-delay response rather than the amplitude response of S_{21}.

Using the normalized g values or the normalized group-delay values of Table I, the required group-delay values can be calculated. These are listed below in Table II along with the coupling values.

The actual filter is symmetrical (although the low-pass prototype is not, since it is even degree) so that the group-delay values are the same when measured from either end. For the low-pass prototype, the group-delay values from the output end will be identical to the input-end values if g_0 is replaced with g_{n+1}.

The filter was realized in combline form using round rods with the input and output connections made by tapping onto the first and last resonators at an appropriate height along each rod. Tuning screws were placed between each resonator and on the input and output couplings as well as on each resonator so that all couplings and resonant frequencies could be set precisely.

Using the calibrated network analyzer, the group delay of S_{11} for the input and output resonators was set to 18.5 ns at 2.3 GHz. Resonator 6 was then shorted and the tuning process started at resonator 1. The basic steps are as follows.

1) Short all resonators except resonator 1. (Precise results are obtained only if the resonators are properly shorted rather than simply detuned.)
2) Adjust resonator 1 and the input coupling to set the specified group delay (18.5 ns).
3) Tune the second resonator and the coupling between resonators to get a symmetrical group-delay response about the center frequency (2.3 GHz) and with the specified value (32.2 ns). To maintain symmetry, it may be necessary to readjust resonator 1 if the coupling is sufficiently strong to "detune" the resonator.
4) Progress through the filter, tuning each resonator in turn and maintain symmetry of the group-delay response by trimming the prior resonator if necessary.
5) When the last resonator is reached (and the filter output is properly terminated), observe the amplitude response of S_{11} and tune the last resonator and the final inter-resonator coupling screw to get the specified return loss.

In principle, the final resonator tuning can be done with a short circuit (for even number of resonators) or an open circuit (for an odd number of resonators) on the output to set the specified group delay. However, the short- or open-circuit plane position is somewhat indeterminate and the matched output gives a much more sensitive response.

The above steps were carried out for the six-section combline filter. Apart from the tuning to maintain group-delay symmetry as each resonator was tuned, no iterative tuning was done and the response obtained after the sequential tuning matched the predicted response very well. Only one of the six return-loss nulls was not sharply defined (indicating a slight mistuning of a resonator), although the finite loss of the filter completely masks the passband ripple. The comparative results are shown in Table III.

The group-delay peaks of S_{21} were predicted to occur at a spacing of 30.5 MHz and the measured spacing was also 30.5 MHz within the experimental error.

TABLE IV
SYMMETRICAL CHEBYSHEV AND APPROXIMATE ELLIPTIC PROTOTYPE VALUES AND MEASURED COUPLING VALUES

Chebychev	Elliptic	Theory Q_E/Coupling Values		Measured (Frequency Crossing)	Measured (Group Delay)
.8072	.8072	Q_{E1}	33.8	--	32.5
1.413	1.413	k_{12}	.0223	.0232	.0228
1.7824	1.7824	k_{23}	.0150	.0151	.0150
1.6833	1.6833	k_{34}	.0137	.0136	.0136
J_4 = .953	1.251	k_{45}	.0177	.0162	.0162
1.6833	1.6833	k_{56}	.0137	.0134	.0135
1.7824	1.7824	k_{67}	.0150	.0150	.0148
1.413	1.413	k_{78}	.0223	.0223	.0222
.8072	.8072	Q_{E8}	33.8	--	31.8
J_3 = 0	- .25	k_{36}	- .00335	- .00298	- .00295

TABLE V
MEASURED GROUP-DELAY VALUES FOR VARIOUS RESONATOR COMBINATIONS

Tuned Resonator	Group Delay (ns)	Tuned Resonator	Group Delay (ns)
1	23.2	8	23.4
1, 2	42.7	8, 7	45.8
1, 2, 3	77.9	8, 7, 6	75
1, 2, 3, 4	94.4	8, 7, 6, 5	102
1, 2, 3, 4, 5	117	8, 7, 6, 5, 4	110.5
1, 2, 3, 6	1190	8, 7, 6, 3	1180
1, 2, 3, 6, 7	80.4		
1, 2, 3, 6, 5	81		

The coupling values were values measured using the frequency-crossing method as each resonator was tuned. These measured values of coupling are listed in Table II. The very close agreement verifies the sensitivity and accuracy of the group-delay method for setting coupling values.

The method can also be applied (with due care) to the multicoupled-resonator filters that are commonly used for elliptic- and linear-phase filters. An $n = 8$ elliptic filter with a single cross coupling was designed using the perturbation method of Levy [7]. The initial prototype was a 0.01-dB ripple Chebyshev filter with a nominal bandwidth of 21 MHz and center frequency of 880 MHz. The transmission nulls were nominally placed at ±16 MHz from the center frequency by appropriate cross coupling between resonators 3 and 6. Following Levy, the symmetrical Chebyshev low-pass prototype values, the modified values for the elliptic response, and the coupling values are shown in Table IV.

The filter was constructed using coaxial resonators with loop coupling and capacitive probe coupling for the elliptic cross coupling. Unlike the previous example, all the coupling values were fixed with only the resonant frequency of the resonators being able to be adjusted. The objective here is to compare the measuring techniques rather than to specifically compare theory with measurement.

The coupling values were measured using the frequency-crossing method and the group-delay technique. Since each theory is only applicable for in-line coupling, the coupling values can be calculated only when this situation applies. To calculate k_{36}, for example, resonators 4 and 5 must be completely shorted and the resonators tuned are 1, 2, 3, and 6. For improved accuracy, coupling measurements are made from both ends of the filter. Thus, k_{76} calculated with the signal applied to resonator 8 will be more accurate than if measured through resonators 1–6.

The measured data for the group-delay technique is shown in Table V.

Where a value was measured from different ends (e.g., k_{36}, k_{45}) the average result is shown in Table IV. To measure k_{36} using the delay method, the frequency span of the network analyzer was reduced from 50 to 5 MHz to enable the very sharp peak to be accurately measured. It is interesting to note that when resonator 7 was tuned after 1–3, and 6, the very high group delay of 1190 ns measured for resonators 1–3, and 6 reduced to 80.4 ns. This value gave a result of $k_{67} = 0.0135$, but the correct result of 0.0148 would have been obtained if the measured group delay had been 80 ns. The method is very sensitive in this case since two nearly equal values (77.9 and 80.4 ns) are being subtracted to calculate the coupling. This simply verifies the expected result that the accuracy of subsequent coupling values would be low when measured through a coupling value that is considerably smaller. This is not an issue in most filters, as it is only the cross couplings that differ appreciably from the other couplings and there is no necessity to measure other couplings via cross coupling. Table III shows that the coupling values measured by the group-delay technique agree very closely with those determined by the frequency-crossing method, the errors being less than ±1%.

IV. Effect of Finite Q

The above procedure is strictly correct only for lossless (i.e., infinite) Q resonators. Finite Q can be incorporated by using the following complex low-pass-to-bandpass transformation:

$$\omega^1 \rightarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) - \frac{j\omega_0}{Q_u(\omega_2 - \omega_1)}. \quad (14)$$

Note that $\partial\omega^1/\partial\omega$ is the same as for the lossless transformation.

For a single resonator, ω^1 from (14) is substituted into (7) to give

$$\Gamma_d(\omega_0) = \frac{4g_1g_0}{(\omega_2 - \omega_1)\left[1 - \left(\frac{\omega_0}{Q_u(\omega_2 - \omega_1)}\right)^2 g_0^2 g_1^2\right]} = \frac{4g_1g_0}{(\omega_2 - \omega_1)\left[1 - \left(\frac{Q_E}{Q_u}\right)^2\right]}. \quad (15)$$

In terms of the bandpass equivalent circuit, the following values are readily derived:

$$\Gamma_d(\omega_0) = \frac{4Z_0C}{1 - \left(\frac{Z_0}{R}\right)^2} \quad (16)$$

for a single-shunt resonator and

$$\Gamma_d(\omega_0) = \frac{4Z_0L}{Z_0^2 - R^2} \quad (17)$$

for a single-series resonator where $Q_u = \omega_0CR$ for a shunt resonator and

$$Q_u = \frac{\omega_0L}{R}$$

for a series resonator.

Equation (15) can be rearranged to give

$$\frac{Q_E}{1 - \left(\frac{Q_E}{Q_u}\right)^2} = \frac{\omega_0\Gamma_d(\omega_0)}{4} \quad (18)$$

and

$$\frac{Q_E}{Q_u} = \frac{1 - |S_{11}|}{1 + |S_{11}|}. \quad (19)$$

Therefore, Q_E/Q_u can be determined from the magnitude of S_{11} at the resonant frequency, thus allowing Q_E to be determined from the group delay. For a single resonator, the effect of loss is to increase the measured group delay compared to the case when the resonator Q is infinite.

TABLE VI

Q_u	Γ_d (Empirical) (ns)	Γ_d (Calculated) (ns)
∞	37.8	37.7
450	35.3	35.1
165	18.0	18.3

The reflection coefficient group delay for two coupled lossy resonators is then

$$\Gamma_{d2}(\omega_0) = \frac{4g_2}{(\omega_2 - \omega_1)}\left[\frac{1 - a^2g_1g_2}{g_0(1 + a^2g_1g_2)^2 - a^2g_2^2}\right] \quad (20)$$

where

$$a = \frac{\omega_0}{Q_u(\omega_2 - \omega_1)} = \left(\frac{Q_E}{Q_u}\right)\frac{1}{g_1g_0}$$

so

$$k_{12}^1 = k_{12}\left[1 - \left(\frac{Q_E}{Q_u}\right)^2\right]^{1/2} \frac{\left[g_0\left(1 + \frac{1}{(Q_uk_{12})^2}\right)^2 \frac{g_2}{g_1}\right]}{\left[1 - \frac{1}{(Q_uk_{12})^2}\right]} \quad (21)$$

where

$$k_{12} = \frac{4}{(\omega_0)(\Gamma_{d_1}\Gamma_{d_2})^{1/2}}.$$

For an odd number of resonators, the finite Q increases the measured group delay, whereas for an even number it decreases the group delay compared to the lossless case. If the coupling value is defined by

$$k_{i,i+1} = \frac{4}{\omega_0[(\Gamma_{d_{i+1}} - \Gamma_{d_{i-1}})(\Gamma_{d_i} - \Gamma_{d_{i-2}})]^{1/2}} \quad (22)$$

then, in general, $k_{i,i+1}$ will increase if i is odd and decrease if i is even as the Q_u of the resonators decreases. The effect of loss on the reflection group delay is to increase the maximum and reduce the minimum values of the group-delay response. This is illustrated in Fig. 4, which shows the group-delay response for two resonators. The curves were derived using a commercial circuit-analysis program and modeling the first two resonators of the combline filter described in Example 1 by LC–R elements. The bandwidth in this case was set at 23 MHz, which for a value of Q_E/Q_u of 0.175 gives a resonator Q_u of 450. The empirically derived group-delay values at ω_0 and the calculated ones are shown in Table VI for $Q_u = \infty$, 450, and 165.

If a 23-MHz bandwidth filter with six resonators and a resonator Q_u of 450 was realized at 2.3 GHz, the insertion loss would be over 7 dB. If the Q_u was only 165, the insertion loss would be nearly 20 dB. This generally confirms the validity

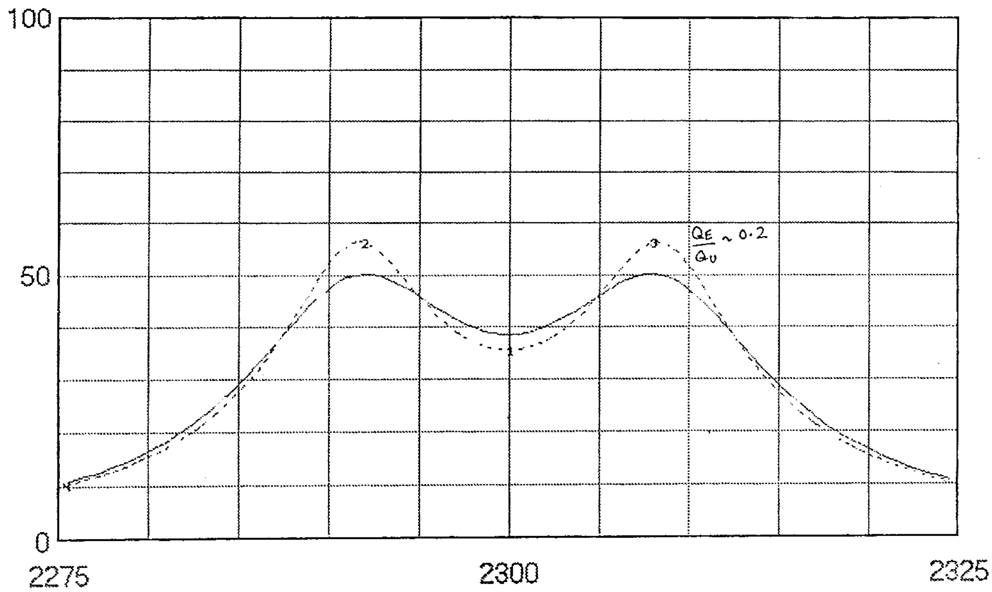

Fig. 4. Group-delay response for two resonators with infinite Q_u and finite Q_u.

of the assumption that loss can be neglected in calculating Γ_d for most practical filters where insertion losses of less than 3 dB are typical.

The effect of finite Q will be small enough to be neglected provided $(Q_E/Q_u) < 0.1$ or $Q_u k > 10$. This condition is usually readily met by most practical filter structures, except perhaps by cross-coupled filters using very low cross-coupling values.

V. Bandwidth Limitations

The coupled-resonator technique requires that coupling reactances are frequency invariant, the resonators can be represented as *LC* circuits, and that parasitic characteristics such as coupling between nonadjacent resonators is negligible. These conditions all become less valid as the bandwidth of the filter is increased. The discrepancy between the actual response and that obtained from setting group-delay values will depend on the physical configuration of the filter as well as the relative bandwidth. The theory was tested for a 20% bandwidth interdigital filter at 3.9 GHz. The actual bandwidth of the filter determined from the S_{21} response after setting the group-delay values was very close (within 2%) to the theoretical value. However, the return loss was only about 22 dB (instead of 26 dB) after the initial tuning and some further iterative tuning was necessary. This may be partly due to nonadjacent resonator coupling although progressive measurements of the coupling coefficients showed that the values changed by less than 0.5% indicating nonadjacent coupling is negligible. With wider bandwidth filters, the group delay of S_{11} becomes quite small and allowance may have to be made for propagation delays if the filter is physically large. The equivalent circuit model is a lumped element one and does not account for the time for the energy to travel across the resonator. If the filter bandwidth is wide enough to cause significant errors in the design, then this can usually be resolved by tuning a prototype filter for the optimum response, measuring the group-delay values and setting these in subsequent filters.

VI. Conclusion

The theory of filter design using low-pass prototypes and low-pass-to-bandpass transformations has been extended to include the group delay of the input reflected signal. The simple equations relating the group delay at the center frequency to the low-pass prototype values and to the coupling coefficients for inverter coupled filters have been derived. This information can be used to measure coupling coefficients and to precisely tune filters to achieve a specific response without iterative tuning.

With the measurement capability of vector network analyzers, the results of the method both in measuring coupling coefficients and in tuning filters have been shown to be quite good. The effects of loss have also been included and have been shown to be negligible for most practical filters. While the analysis has been derived only for “in-line” filters, the technique can be applied to cross-coupled filters with due care. The method is also very useful for setting the component values of *LC* filters, but particular attention must be given to circuit parasitics as these may change the format of the equivalent circuit. The effect of finite Q has been discussed. This may be required for narrow bandwidth filters using relatively low Q resonators or when very small cross couplings are used for linear-phase or elliptic-response filters. Finally, it can be noted that the group delay of the reflected signal contains the same information as the coupling coefficient. Since the group delay can be directly measured, it may be more appropriate to consider filter design in terms of time rather than coupling coefficients.

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REFERENCES

[1] G. Mathaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance Matching Networks and Coupling Structures*. Norwood, MA: Artech House, 1980.
[2] M. Dishal, "Alignment and adjustment of synchronously tuned multiple resonant circuit filters," *Proc IRE*, vol. 30, pp. 1448–1455, Nov. 1951.
[3] N. McDonald, "Measurements of intercavity coupling," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, p. 162, Mar. 1976.
[4] A. Atia and A. Williams, "Measurement of intercavity coupling," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 519–522, June 1975.
[5] M. H. Chen, "Short circuit tuning method for singly terminated filter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 1032–1036, Dec. 1977.
[6] A. Williams, R. Egri, and R. Johnson, "Automatic measurement of filter coupling parameters," in *Proc. IEEE MTT-S Symp. Dig.*, Boston, MA, May 1983, pp. 418–420.
[7] R. Levy, "Filter with single transmission zeros at real or imaginary frequencies," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 172–181, Apr. 1976.

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